| SEMESTER | IV | QP CODE | 23MAT41 |
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**P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA**

**SEM END EXAMINATIONS APRIL -2025**

**II B.SC : MATHS:RING THEORY**

**TIME: 2 HRS**

| DATE&SESSION | 07.04.2025 & AN | REG NO | 2 | 3 | 2 | M | A | T | 2 | 9 | MAX MARKS | 50 |
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### SECTION-I

**Answer any THREE of the following questions. And attempt one question from Each section part Each question carries TEN marks** **3X10=30Marks**

### PART-A

1. Prove that $Q(\sqrt{2}) = \{ a + b\sqrt{2} \,|a, b \in Q\}$ is a field with respect to ordinary addition and multiplication of numbers
2. If $U_1, U_2$ are two ideals of a ring R then $U_1 \cup U_2$ is an ideal of R if and only if one is containing to the other one
3. If U is an ideal of a ring R then prove that the set $\frac{R}{U} = \{ x + U \,|x \in R\}$ is a ring with respect to the induced operations of addition $(+)$ and multiplication $(\,.\,)$ of cosets defined by
$(a + U) + (b + U) = (a + b) + U$ and $(a + U).(b + U) = ab + U$ for
$a + U, b + U \in \frac{R}{U}$

### PART-B

4. State and prove fundamental theorem of homomorphism
5. If M is a maximal ideal of the ring of integers Z then show that M is generated by prime integers
6. If $F$ is a field then prove that $F(x)$ is a principal ideal domain

### SECTION-II

**Answer any FOUR of the following questions. Each question carries FIVE marks**

**4 X 5=20Marks**

7. Show that a field has no zero divisors
8. Show that the ideals of a field $F$ are only $\{0\}$ and $F$ itself
9. The intersection of two ideals of a ring R is an ideal of R
10. If U is an ideal of the ring R and $a, b \in R$ then prove that $a + U = b + U \Leftrightarrow a - b \in U$
11. Prove that every homomorphic image of a ring is a ring
12. Prove that $f(x) = 25x^5 - 9x^4 + 3x^2 - 12 \in Z[x]$ is irreducible over Q
13. State and prove the Remainder theorem